On diagonal cycle systems

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Abstract

A diagonal cycle is a cycle of length $2n$ together with one edge between two vertices at distance n in the cycle. For example, $K_4 - e$ is a cycle of length four with one extra edge between two vertices at distance two in the cycle. Methods for decomposing the complete graph K_v into diagonal cycles are given when $v \equiv 0 \pmod{2n+1}$ if n is odd, and when $v \equiv 1 \pmod{2n+1}$, if $n \equiv 3 \pmod 4$; in the case $n \equiv 1 \pmod 4$ further partial results are obtained.

1 Introduction and necessary conditions

The necessary and sufficient conditions for the existence of a decomposition of the complete graph on v vertices, K_v, into cycles of length n are now known. There have been many papers written on the properties of cycle systems. For a survey of cycle systems see [6]. Until recently, the existence of some cycle sytems was not known. These cases are covered in [2] and [7]. A cycle of length $2n$ ($n \geq 2$) is a graph on the vertices $a_1, a_2, \ldots, a_{2n}$ with edge set $\{a_i, a_{i+1}\}$, $1 \leq i \leq 2n-1$, $\{a_1, a_{2n}\}$. A *diagonal edge* adjoined to such a cycle is any edge between two vertices at distance n apart around the cycle. In the above cycle, if a diagonal edge such as $\{a_1, a_{n+1}\}$ is adjoined to the cycle, we call the resulting graph a *diagonal cycle* and denote it by $[a_1, a_2, \ldots, a_n; a_{n+1}, a_{n+2}, \ldots, a_{2n}]$. Furthermore, we denote any such diagonal cycle by D_{2n}. Thus, for instance, $K_4 - e$ is a diagonal cycle of the smallest possible size, denoted by D_4. There have also been many papers written on the subject of $K_4 - e$ designs. The necessary and sufficient conditions for the existence of $K_4 - e$ designs are given in [3], the intersection problem is solved in [4] and blocking sets are investigated in [5].

A decomposition of a graph into edge-disjoint copies of D_{2n} is called a D_{2n}-*decomposition*. A D_{2n}-decomposition of the complete graph on v vertices is also called a D_{2n}-*design of order* v. A D_{2n}-decomposition of a graph with vertex set V may be written as a pair (V, B), where B is a collection of copies of D_{2n} that partitions the edge set of the graph.

The graph D_{2n} has $2n+1$ edges, so the expected spectrum (or the set of possible orders) of D_{2n}-designs will consist of all v for which

$$\frac{v(v-1)}{2(2n+1)}$$

is an integer. When $2n+1 = p^a$, where p is prime and a is an integer, the expected spectrum for D_{2n}-designs is

$$v \equiv 0 \text{ or } 1 \pmod{2n+1}. \qquad (*)$$

When $2n+1 \neq p^a$, the set of v satisfying $(*)$ will form a subset of the expected spectrum for D_{2n}-designs.

In this paper we demonstrate methods for constructing D_{2n}-designs of order v where n is odd and v satisfies $(*)$, except when $v \equiv 2n+2 \pmod{4n+2}$ where $n \equiv 1 \pmod 4$ and $n \geq 13$. Thus, this paper gives the necessary and some sufficient conditions for the existence of certain G-designs, where G can be thought of as a generalization of $K_4 - e$ or an extension of an even length cycle.

2 D_6-designs

In this section we show that for all $v \equiv 0$ or $1 \pmod 7$, $v > 7$, there is a D_6-design of order v.

LEMMA 2.1 *There is no D_6-design of order 7.*

Proof
In any D_6-design of order 7 we expect 3 copies of D_6. Let the vertex set of K_7 be $V = \{1,2,3,4,5,6,7\}$. Without loss of generality, we can label one of the expected copies of D_6 as $[1,2,3;4,5,6]$. (See Figure 1a.) This forces the vertices shown below in Figures 1b and 1c on the remaining two expected copies of D_6.

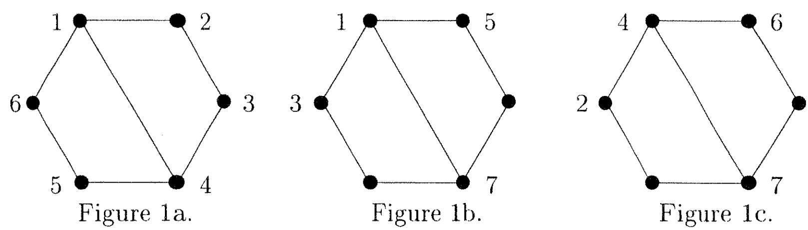

Figure 1a. Expected Copy 1
Figure 1b. Expected Copy 2
Figure 1c. Expected Copy 3

This leaves nowhere for the edges $\{3,5\}$ or $\{2,6\}$. Hence there is no D_6-design of order 7. □

We now present examples of D_6-decompositions of graphs required for our constructions.

EXAMPLE 1
Let $V = \mathbb{Z}_8$ and let $B = \{[0,1,7;4,5,3] + i : i = 0,1,2,3\}$.

Then (V, B) is a D_6-design of order 8.

EXAMPLE 2
Let $V = \mathbb{Z}_{14}$ and let

$$\begin{aligned} B = \{ & [0,1,2;4,5,7], && [0,2,9;5,8,3], && [0,6,3;10,2,12], && [1,5,2;11,7,10], \\ & [1,8,9;13,2,3], && [1,12,6;9,11,4], && [2,7,1;6,10,8], && [3,9,12;5,6,4], \\ & [4,8,6;7,9,10], && [4,9,0;13,3,12], && [5,10,12;13,6,11], && [8,13,10;11,3,7], \\ & [11,13,7;12,8,0] \}. \end{aligned}$$

Then (V, B) is a D_6-design of order 14.

EXAMPLE 3
Let $V = \mathbb{Z}_3 \times \mathbb{Z}_7$ and let B contain the blocks arising from the following set, with the first components all cycled modulo 3, and the second components fixed.

$$\begin{aligned} \{ & [(0,0),(0,2),(0,4);(0,5),(2,5),(1,2)], && [(0,0),(2,0),(2,1);(1,3),(1,2),(2,6)], \\ & [(0,1),(0,6),(1,3);(0,2),(2,4),(2,1)], && [(0,2),(2,1),(1,2);(0,5),(1,6),(1,0)], \\ & [(0,3),(1,4),(0,1);(1,0),(0,5),(2,3)], && [(0,3),(2,1),(1,0);(2,5),(2,6),(1,6)], \\ & [(0,4),(1,4),(2,1);(2,5),(1,6),(2,2)], && [(1,1),(1,3),(1,0);(2,6),(1,4),(2,5)], \\ & [(2,0),(0,4),(0,3);(2,4),(1,6),(1,4)], && [(2,3),(2,5),(0,1);(2,6),(2,2),(0,2)] \}. \end{aligned}$$

Then (V, B) is a D_6-design of order 21.

EXAMPLE 4
Let $V = \mathbb{Z}_7 \times \mathbb{Z}_2$ and let $B = \{[(0,0),(1,1),(3,0);(0,1),(1,0),(3,1)] + (i,0) : i \in \mathbb{Z}_7\}$.

Then (V, B) is a D_6-decomposition of $K_{7,7}$, where V is partitioned in the obvious way.

The following three lemmas use the D_6-designs given in Examples 1, 2, 3 and 4 to construct D_6-designs of order v for all $v \equiv 0$ or $1 \pmod 7$, $v > 7$.

LEMMA 2.2 *There is a D_6-design of order $7k+1$, $k \geq 1$.*

Proof
Let $V = (\mathbb{Z}_k \times \mathbb{Z}_7) \cup \{\infty\}$ and let B contain the copies of D_6 from the following two types of D_6-decompositions.

Type 1: For each i, $0 \leq i \leq k-1$, place a D_6-design of order 8 on $(\{i\} \times \mathbb{Z}_7) \cup \{\infty\}$.
Type 2: For each i and j satisfying $0 \leq i < j \leq k-1$, place a D_6-decomposition of $K_{7,7}$ on $\{i,j\} \times \mathbb{Z}_7$.

Then (V, B) is a D_6-design of order $7k+1$. □

LEMMA 2.3 *There is a D_6-design of order $14k$, $k \geq 1$.*

Proof
Let $V = \mathbb{Z}_{2k} \times \mathbb{Z}_7$ and let B contain the copies of D_6 from the following two types of D_6-decompositions.

Type 1: For each i, $0 \leq i \leq k-1$, place a D_6-design of order 14 on $\{2i, 2i+1\} \times \mathbb{Z}_7$.
Type 2: For each i and j satisfying $0 \leq i < j \leq 2k-1$ and $(i,j) \neq (2x, 2x+1)$ for any x, $0 \leq x \leq k-1$, place a D_6-decomposition of $K_{7,7}$ on $\{i,j\} \times \mathbb{Z}_7$.

Then (V, B) is a D_6-design of order $14k$. □

LEMMA 2.4 *There is a D_6-design of order $14k+7$, $k \geq 1$.*

Proof
Let $V = \mathbb{Z}_{2k+1} \times \mathbb{Z}_7$ and let B contain the copies of D_6 from the following three types of D_6-decompositions.

Type 1: Place a D_6-design of order 21 on $\mathbb{Z}_3 \times \mathbb{Z}_7$.
Type 2: For each i, $2 \leq i \leq k$, place a D_6-design of order 14 on $\{2i-1, 2i\} \times \mathbb{Z}_7$.
Type 3: For each i and j satisfying $0 \leq i < j \leq 2k$, $(i,j) \neq (2x-1, 2x)$ for any x, $2 \leq x \leq k$, and $\{i,j\} \not\subset \{0,1,2\}$, place a D_6-decomposition of $K_{7,7}$ on $\{i,j\} \times \mathbb{Z}_7$.

Then (V, B) is a D_6-design of order $14k+7$. □

3 General Cases

In this section we give methods for constructing the D_{2n}-decompositions which will be used in the constructions in Section 5. In Figures 2 and 3 the heavy edge denotes the diagonal edge of the diagonal cycle.

LEMMA 3.1 *There is a D_{2n}-design of order $2n+1$, where n is odd and $n \geq 5$.*

Proof
If $n = 5$, let $V = (\mathbb{Z}_5 \times \mathbb{Z}_2) \cup \{\infty\}$ and let $B = \{[(1,0),(0,0),(0,1),(4,0),(1,1);(3,0),(2,1),(4,1),(3,1),\infty] + (i,0) : i \in \mathbb{Z}_5\}$. If $n = 7$, see Example 5 below. If $n = 9$, let $V = (\mathbb{Z}_9 \times \mathbb{Z}_2) \cup \{\infty\}$ and let $B = \{[(1,0),(0,0),(0,1),(8,0),(1,1),(7,0),(2,1),(6,0),(3,1);(5,0),(4,1),(8,1),(5,1),(7,1),(6,1),\infty,(2,0),(4,0)] + (i,0) : i \in \mathbb{Z}_9\}$. If $n \geq 11$, let $V = (\mathbb{Z}_n \times \mathbb{Z}_2) \cup \{\infty\}$ and let

$$B = \{[(1,0),(0,0),(0,1),(n-1,0),(1,1),(n-2,0),(2,1),\ldots,(n-m,0),(m,1),\ldots,$$
$$(\frac{n+3}{2},0)(\frac{n-3}{2},1);(\frac{n+1}{2},0),(\frac{n-1}{2},1),(n-1,1),(\frac{n+1}{2},1),(n-2,1),$$
$$(\frac{n+3}{2},1),\ldots,(n-m,1),(\frac{n+(2m-1)}{2},1),\ldots,(\lfloor\frac{3n}{4}\rfloor,1),\infty,(n \pmod 4)+$$
$$\lfloor\frac{n-5}{4}\rfloor,0),\ldots,(m,0),(\frac{n-(2m-3)}{2},0),\ldots,(2,0),(\frac{n-1}{2},0)] + (i,0) : i \in \mathbb{Z}_n\}.$$

Then a straightforward check shows that (V, B) is a D_{2n}-design of order $2n+1$. □

EXAMPLE 5

Let $V = (\mathbb{Z}_7 \times \mathbb{Z}_2) \cup \{\infty\}$ and let $B = \{[(1,0),(0,0),(0,1),(6,0),(1,1),(5,0),(2,1);$ $(4,0),(3,1),(6,1),(4,1),(5,1),\infty,(3,0)] + (i,0) : i \in \mathbb{Z}_7\}$.

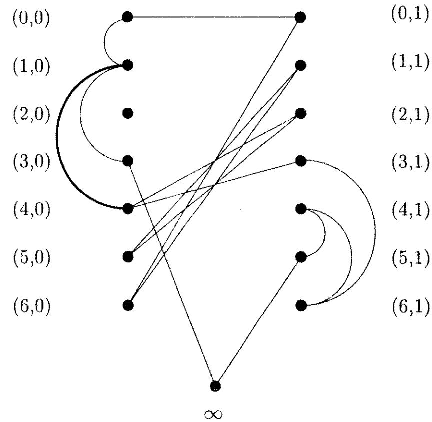

Figure 2. The starter configuration for a D_{14}-design of order 15.

Then (V, B) is a D_{14}-design of order 15.

LEMMA 3.2 *There is a D_{2n}-design of order $2n+2$, where $n \equiv 3 \pmod 4$.*

Proof

If $n = 3$ see Example 1, otherwise let $V = \mathbb{Z}_{2n+2}$, where $n \equiv 3 \pmod 4$. Define a sequence of differences, $\{d_j\}$, in the following way:
$\{d_j\} = \langle 1, -2, 3, -4, \ldots, 2m-1, -2m, \ldots, \frac{n-1}{2}, -(\frac{n+1}{2}), \frac{n+5}{2}, -(\frac{n+7}{2}), \frac{n+9}{2}, -(\frac{n+11}{2}),$ $\ldots, \frac{n+(4m+1)}{2}, -(\frac{n+(4m+3)}{2}), \ldots, n-1, -n, \frac{n+3}{2}\rangle$.

The sequence of differences, $\{d_j\}$, forms a 'semi-cycle' in the starter configuration defined below. By 'semi-cycle' we mean the path of length n on the vertices $a_1, a_2, \ldots, a_{n+1}$ or $a_{n+1}, a_{n+2}, \ldots, a_{2n}$. Let $B = \{[0, \sum_{j=1}^{1} d_j, \sum_{j=1}^{2} d_j, \ldots, \sum_{j=1}^{n-1} d_j;$ $n+1, n+1+\sum_{j=1}^{1} d_j, n+1+\sum_{j=1}^{2} d_j, \ldots, n+1+\sum_{j=1}^{n-1} d_j] + i : i = 0, 1, \ldots, n\}$.

Then (V, B) is a D_{2n}-design of order $2n+2$. □

EXAMPLE 6

Let $V = \mathbb{Z}_{16}$ and let $\{d_j\} = \langle 1, -2, 3, -4, 6, -7, 5\rangle$.
Let $B = \{[0, 1, 15, 2, 14, 4, 13; 8, 9, 7, 10, 6, 12, 5] + i : i = 0, 1, \ldots, 7\}$.

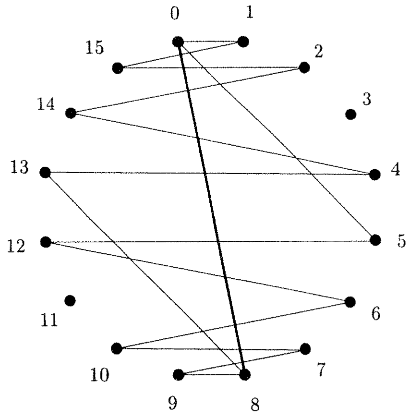

Figure 3. The starter configuration for a D_{14}-design of order 16.

Then (V, B) is a D_{14}-design of order 16.

LEMMA 3.3 *There is a D_{2n}-design of order $4n + 3$, where $n \equiv 1 \pmod 4$.*

Proof
Let $V = \mathbb{Z}_{4n+3}$, where $n \equiv 1 \pmod 4$. Define a sequence of differences, $\{d_j\}$, in the following way:
$\{d_j\} = \langle -2, 4, -6, 8, \ldots, -(n-3), (n-1), -(n+3), n+5, -(n+7), n+9, \ldots,$
$2n-4, -(2n-2), 1, 2n+1, 2n-1, -(2n-3), 2n-5, -(2n-7), \ldots, 5, -3\rangle$.
Once again, the sequence of differences, $\{d_j\}$, forms a 'semi-cycle' in the starter configuration defined below. Let $B = \{[\sum_{j=1}^{1} d_j, \sum_{j=1}^{2} d_j, \ldots, \sum_{j=1}^{n} d_j; \sum_{j=1}^{n+1} d_j, \sum_{j=1}^{n+2} d_j, \ldots, \sum_{j=1}^{2n} d_j] + i : i \in \mathbb{Z}_{4n+3}\}$.

Then (V, B) is a D_{2n}-design of order $4n + 3$. □

LEMMA 3.4 *There is a D_{2n}-decomposition of $K_{2n+1,2n+1}$, where n is odd.*

Proof
Let $V = \mathbb{Z}_{2n+1} \times \mathbb{Z}_2$ where n is odd. We shall consider the case $n \equiv 3 \pmod 4$ and the case $n \equiv 1 \pmod 4$ separately.

The case $n \equiv 3 \pmod 4$. If $n = 3$, see Example 4, otherwise define a sequence of differences, $\{d_j\}$, in the following way:
$\{d_j\} = \langle 1, 2, 3, 4, \ldots, 2m-1, 2m, \ldots, \frac{n-5}{2}, \frac{n-3}{2}, \frac{n+1}{2}, \frac{n+3}{2}, \ldots, n-1, n, \frac{n-1}{2}\rangle$.
Let $B = \{[(0,0), (\sum_{j=1}^{1}(-1)^{j+1}d_j, 1), (\sum_{j=1}^{2}(-1)^{j+1}d_j, 0), \ldots, (\sum_{j=1}^{n-2}(-1)^{j+1}d_j, 1),$
$(\sum_{j=1}^{n-1}(-1)^{j+1}d_j, 0); (0,1), (\sum_{j=1}^{1}(-1)^{j+1}d_j, 0), (\sum_{j=1}^{2}(-1)^{j+1}d_j, 1), \ldots,$
$(\sum_{j=1}^{n-2}(-1)^{j+1}d_j, 0), (\sum_{j=1}^{n-1}(-1)^{j+1}d_j, 1)] + (i, 0) : i \in \mathbb{Z}_{2n+1}\}$.

Then (V, B) is a D_{2n}-decomposition of $K_{2n+1,2n+1}$, where $n \equiv 3 \pmod 4$.

The case $n \equiv 1 \pmod 4$. Define two sequences of differences, $\{a_j\}$ and $\{b_j\}$, in the following way:

If $n=5$ let $\{a_j\} = \langle 9,1,2,3,4\rangle$ and let $\{b_j\} = \langle 7,8,6,10,5\rangle$. If $n>5$ let $\{a_j\} = \langle \frac{3}{2}(n+1), 1, 2, 3, \ldots, n-1\rangle$ and
$\{b_j\} = \langle n+2, n+3, ..., \frac{1}{2}(3n+1), n+1, 2n, 2n-1, \ldots, \frac{1}{2}(3n+5), n\rangle$.
Let $B = \{[(0,0), (\sum_{j=1}^{1}(-1)^{j+1}a_j, 1), (\sum_{j=1}^{2}(-1)^{j+1}a_j, 0), \ldots, (\sum_{j=1}^{n-2}(-1)^{j+1}a_j, 1), (\sum_{j=1}^{n-1}(-1)^{j+1}a_j, 0); (0,1), (\sum_{j=1}^{1}(-1)^{j+1}b_j, 0), (\sum_{j=1}^{2}(-1)^{j+1}b_j, 1), \ldots, (\sum_{j=1}^{n-2}(-1)^{j+1}b_j, 0), (\sum_{j=1}^{n-1}(-1)^{j+1}b_j, 1)] + (i,0) : i \in \mathbb{Z}_{2n+1}\}$.

Then (V,B) is a D_{2n}-decomposition of $K_{2n+1,2n+1}$, where $n \equiv 1 \pmod 4$. □

4 Some useful D_{2n}-designs of order $2n+2$

In this section we give examples of some D_{2n}-designs of order $2n+2$, where $n \equiv 1 \pmod 4$. These designs enable us to complete the spectrum for D_{2n}-designs when $n=5$ and $n=9$.

EXAMPLE 7
Let $V = \mathbb{Z}_3 \times \mathbb{Z}_4$ and let $B =$
$\{[(0,0), (1,0), (0,1), (2,0), (2,1); (0,2), (1,1), (0,3), (1,2), (1,3)] + (i,0),$
$[(0,1), (1,1), (1,2), (0,0), (2,3); (0,3), (2,2), (0,2), (1,0), (1,3)] + (i,0) : i \in \mathbb{Z}_3\}$.

Then (V,B) is a D_{10}-design of order 12.

EXAMPLE 8
Let $V = \mathbb{Z}_5 \times \mathbb{Z}_4$ and let $B =$
$\{[(0,0), (1,0), (0,1), (3,0), (4,1), (1,1), (1,2), (2,0), (3,3);$
$(2,2), (4,2), (2,1), (0,2), (4,0), (2,3), (3,1), (4,3), (0,3)] + (i,0),$
$[(4,1), (3,2), (3,0), (1,0), (0,3), (1,2), (3,3), (0,2), (4,2);$
$(4,3), (1,3), (4,0), (2,2), (1,1), (2,1), (2,0), (0,1), (2,3)] + (i,0) : i \in \mathbb{Z}_5\}$.

Then (V,B) is a D_{18}-design of order 20.

5 General Constructions

Since Section 2 completely deals with D_6-designs, here we consider D_{2n}-designs where n is odd and $n \geq 5$.

LEMMA 5.1 *There is a D_{2n}-design of order $k(2n+1)$, where n is odd, $n \geq 5$, and $k \geq 1$.*

Proof
Let $V = \mathbb{Z}_k \times \mathbb{Z}_{2n+1}$ and let B contain the diagonal cycles from the following two types of D_{2n}-decompositions.
Type 1: For each i, $0 \leq i \leq k-1$, place a D_{2n}-design of order $2n+1$ on $\{i\} \times \mathbb{Z}_{2n+1}$; such a design exists by Lemma 3.1.

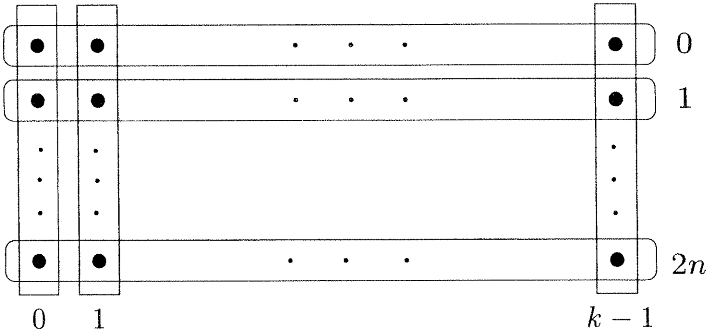

Figure 4. D_{2n}-decompositions of Type 1

Type 2: For each i and j satisfying $0 \le i < j \le k-1$, place a D_{2n}-decomposition of $K_{2n+1,2n+1}$ on $\{i,j\} \times \mathbb{Z}_{2n+1}$; such a decomposition exists by Lemma 3.4.

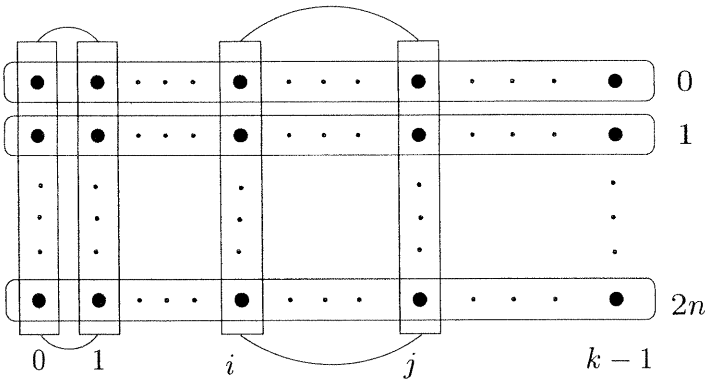

Figure 5. D_{2n}-decompositions of Type 2

Then (V, B) is a D_{2n}-design of order $k(2n+1)$, where n is odd, $n \ge 5$, and $k \ge 1$. □

LEMMA 5.2 *There is a D_{2n}-design of order $k(2n+1)+1$, where $n = 5$, $n = 9$ or $n \equiv 3 \pmod 4$, $n \ge 7$, and $k \ge 1$.*

Proof
Let $V = (\mathbb{Z}_k \times \mathbb{Z}_{2n+1}) \cup \{\infty\}$ and let B contain the diagonal cycles from the following two types of D_{2n}-decompositions.
Type 1: For each i, $0 \le i \le k-1$, place a D_{2n}-design of order $2n+2$ on $(\{i\} \times \mathbb{Z}_{2n+1}) \cup \{\infty\}$; such a design exists by Lemma 3.2, Example 7 or Example 8.
Type 2: For each i and j satisfying $0 \le i < j \le k-1$, place a D_{2n}-decomposition of $K_{2n+1,2n+1}$ on $\{i,j\} \times \mathbb{Z}_{2n+1}$; such a decomposition exists by Lemma 3.4.

Then (V, B) is a D_{2n}-design of order $k(2n+1)+1$, where $n = 5$, $n = 9$ or $n \equiv 3$ (mod 4), $n \ge 7$ and $k \ge 1$.

□

LEMMA 5.3 *There is a D_{2n}-design of order $2k(2n+1)+1$, where $n \equiv 1 \pmod 4$ and $k \geq 1$.*

Proof
Let $V = (\mathbb{Z}_{2k} \times \mathbb{Z}_{2n+1}) \cup \{\infty\}$ and let B contain the diagonal cycles from the following two types of D_{2n}-decompositions.
Type 1: For each i, $0 \leq i \leq 2k-1$, place a D_{2n}-design of order $4n+3$ on $(\{i, i+1\} \times \mathbb{Z}_{2n+1}) \cup \{\infty\}$; such a design exists by Lemma 3.3.
Type 2: For each i and j satisfying $0 \leq i < j \leq 2k-1$, $(i,j) \neq (2x, 2x+1)$, $0 \leq x \leq k-1$, place a D_{2n}-decomposition of $K_{2n+1,2n+1}$ on $\{i,j\} \times \mathbb{Z}_{2n+1}$; such a decomposition exists by Lemma 3.4.

Then (V, B) is a D_{2n}-design of order $2k(2n+1)+1$, where $n \equiv 1 \pmod 4$ and $k \geq 1$.
□

6 Summary

When combined, all of the lemmas in this paper give us the following theorem:

THEOREM 6.1

(i) *When $v \equiv 0 \pmod{2n+1}$ and n is odd, there exists a D_{2n}-design of order v, except when $n = 3$ and $v = 7$.*

(ii)(a) *When $v \equiv 1 \pmod{2n+1}$ and $n = 5$, $n = 9$, or $n \equiv 3 \pmod 4$, there exists a D_{2n}-design of order v.*

(ii)(b) *When $v \equiv 1 \pmod{4n+2}$ and $n \equiv 1 \pmod 4$, $n \geq 13$, there exists a D_{2n}- design of order v. (However if $v \equiv 2n+2 \pmod{4n+2}$, $n \geq 13$, the existence of a D_{2n}-design of order v remains open.)*

Proof
Part (i) follows from Lemmas 2.1 and 5.1.
Part (ii)(a) follows from Lemma 5.2.
Part (ii)(b) follows from Lemma 5.3. □

We conjecture that when $v \equiv 1 \pmod{2n+1}$, a D_{2n}-design exists for all odd n and work is proceeding on this.

Acknowledgements

Many thanks to Elizabeth Billington for suggesting this problem and for her many helpful discussions and suggestions. Thanks also to Peter Adams for use of his graph decomposition package, `autogen` [1]. Examples 2, 3, 7 and 8 were found using `autogen`; the remaining examples were found by hand.

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